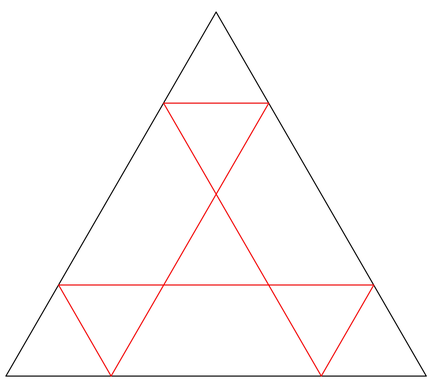
Constantin Kogler, Switzerland

On periodic billiard trajectories in regular polygons and simple closed geodesics on the tetrahedron, cube and octahedron

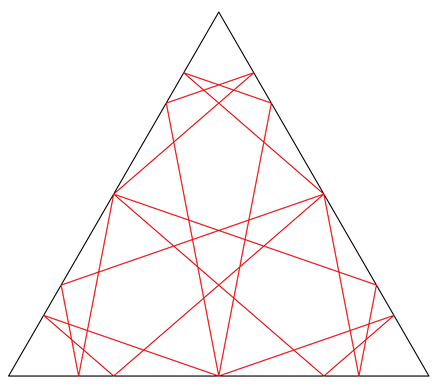
Periodic billiard trajectories in regular polygons

According to Theorem 2, we know a condition for the launching angle of a periodic billiard trajectory for a general regular polygon. Theorem 2 (or a simplification of it) leads to: Equation (1) for the regular triangle and hexagon; Equation (2) for the square; Equation (3) for the regular pentagon and Equation (4) for the regular octagon.

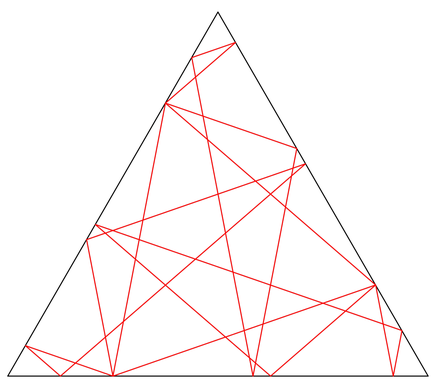
$$\tan(\alpha) = \frac{a_1 \cdot \frac{\sqrt{3}}{2}}{b_0 - \frac{b_1}{2}} \quad (1) \qquad \tan(\alpha) = \frac{a_1}{b_0} \quad (2) \qquad \tan(\alpha) = \frac{a_1 \cdot \sin(\frac{2 \cdot \pi}{5}) + a_2 \cdot \sin(\frac{4 \cdot \pi}{5})}{b_0 + b_1 \cdot \cos(\frac{2 \cdot \pi}{5}) + b_2 \cdot \cos(\frac{4 \cdot \pi}{5})} \quad (3) \qquad \tan(\alpha) = \frac{a_1 \cdot \sin(\frac{\pi}{4}) + a_2}{b_0 + b_1 \cdot \sin(\frac{\pi}{4})} \quad (4)$$



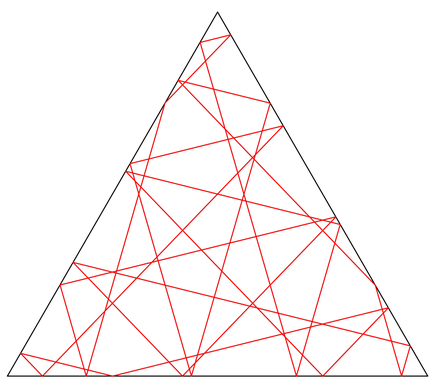
$S^* = 0.25$ and $(a_1, b_0, b_1) = (1, 1, 1)$



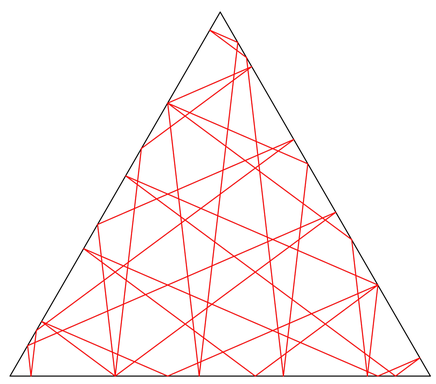
$S^* = 0.25$ and $(a_1, b_0, b_1) = (1, 1, 0)$



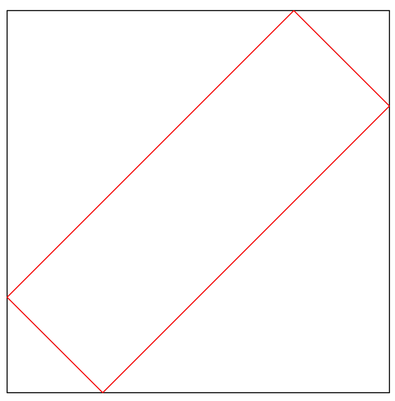
$S^* = 0.25$ and $(a_1, b_0, b_1) = (1, 3, 1)$



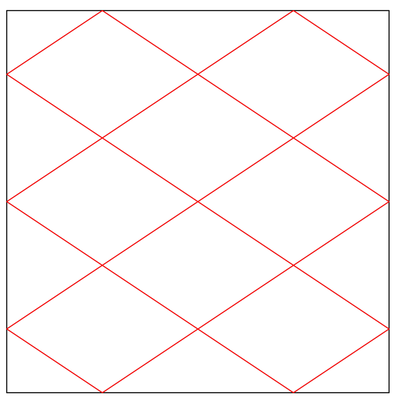
$S^* = 0.25$ and $(a_1, b_0, b_1) = (1, 4, 1)$



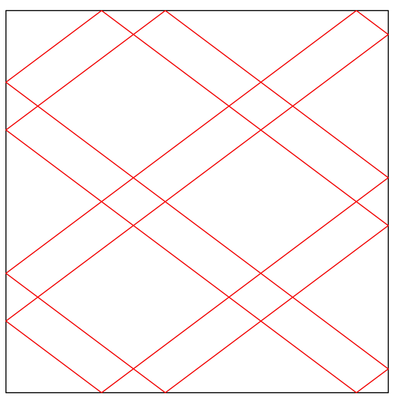
$S^* = 0.25$ and $(a_1, b_0, b_1) = (3, 4, 1)$



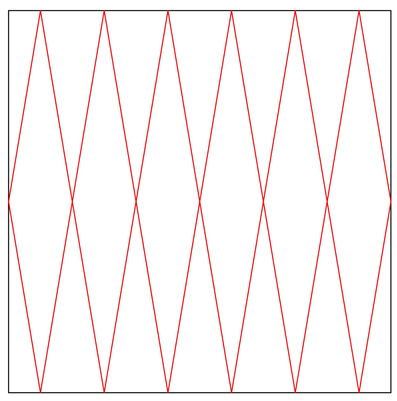
$S^* = 0.25$ and $(a_1, b_0) = (1, 1)$



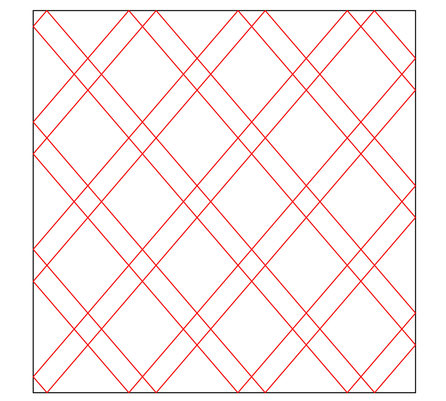
$S^* = 0.25$ and $(a_1, b_0) = (2, 3)$



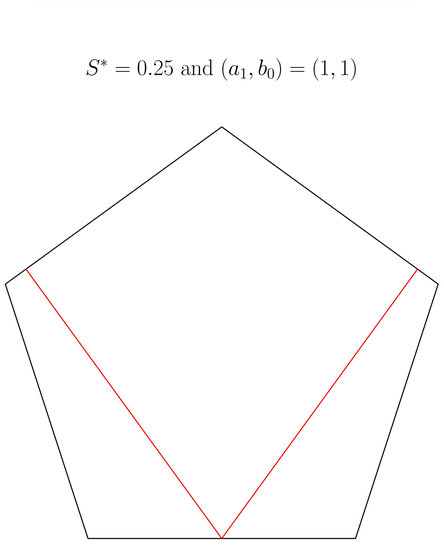
$S^* = 0.25$ and $(a_1, b_0) = (3, 4)$



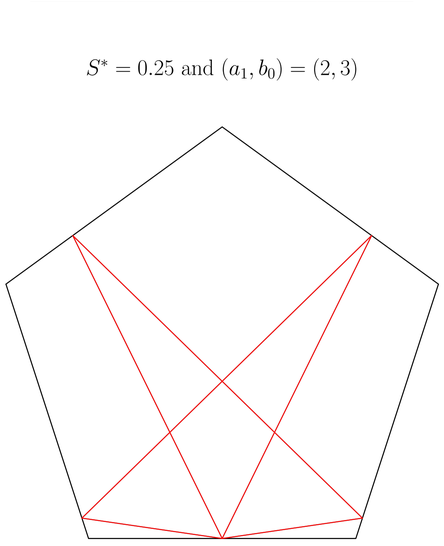
$S^* = 0.25$ and $(a_1, b_0) = (6, 1)$



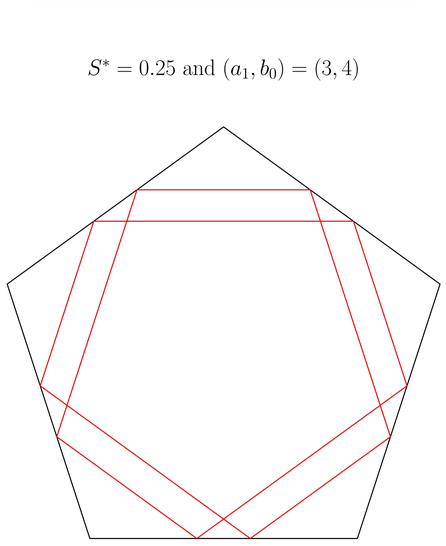
$S^* = 0.25$ and $(a_1, b_0) = (7, 6)$



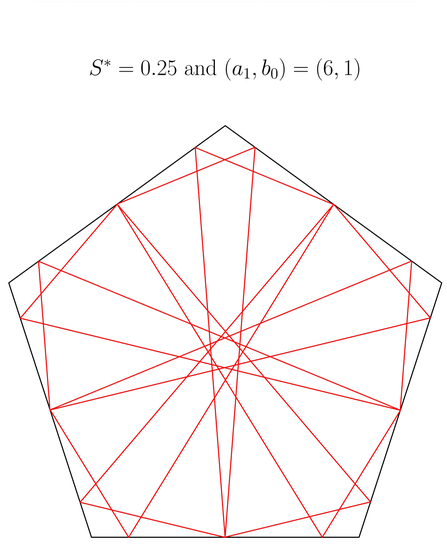
$S^* = 0.5$ and $(a_1, a_2, b_0, b_1, b_2) = (3, 2, 1, -2, 1)$



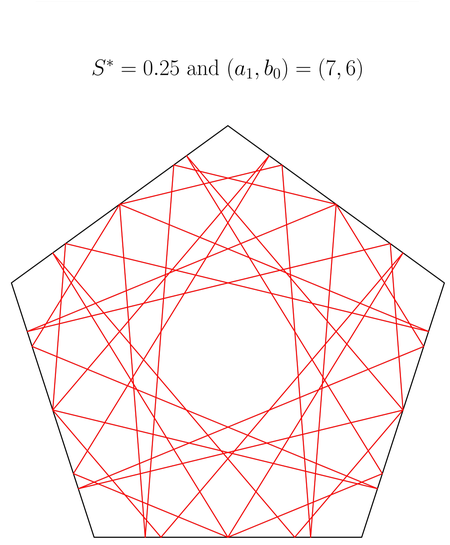
$S^* = 0.5$ and $(a_1, a_2, b_0, b_1, b_2) = (5, 2, 1, -2, 1)$



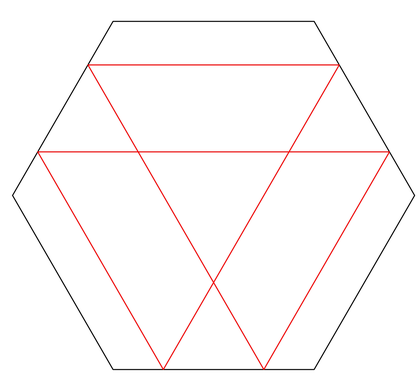
$S^* = 0.4$ and $(a_1, a_2, b_0, b_1, b_2) = (1, 0, 1, 1, 0)$



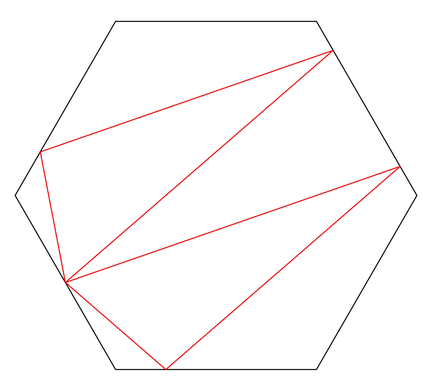
$S^* = 0.5$ and $(a_1, a_2, b_0, b_1, b_2) = (1, 0, 3, 3, 0)$



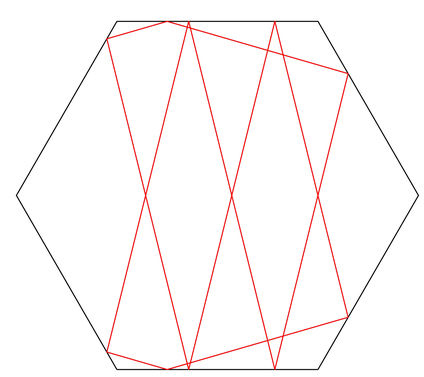
$S^* = 0.5$ and $(a_1, a_2, b_0, b_1, b_2) = (4, 3, 1, 0, -3)$



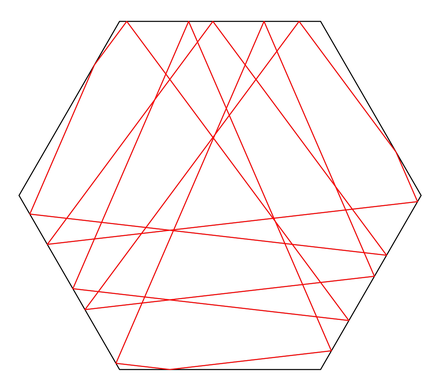
$S^* = 0.25$ and $(a_1, b_0, b_1) = (1, 1, 1)$



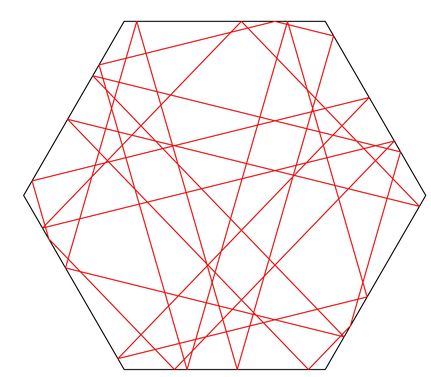
$S^* = 0.25$ and $(a_1, b_0, b_1) = (1, 1, 0)$



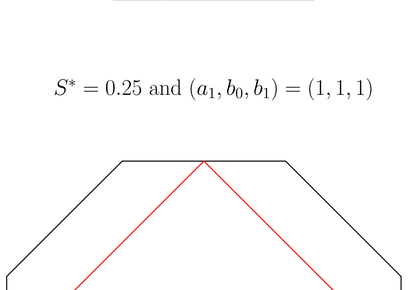
$S^* = 0.25$ and $(a_1, b_0, b_1) = (1, 3, 0)$



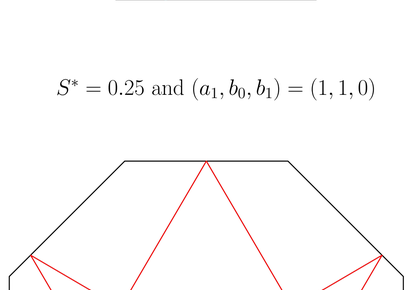
$S^* = 0.25$ and $(a_1, b_0, b_1) = (1, 8, 1)$



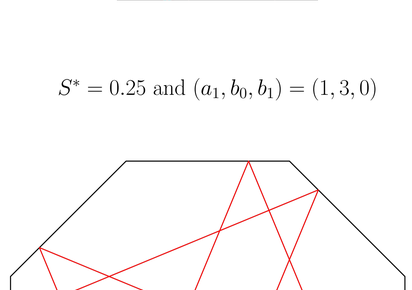
$S^* = 0.25$ and $(a_1, b_0, b_1) = (3, 3, 1)$



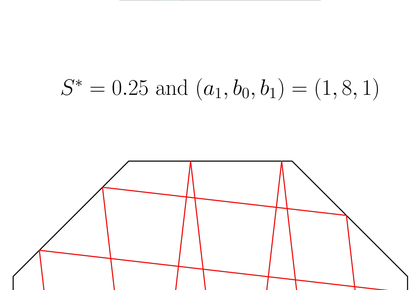
$S^* = 0.5$ and $(a_1, a_2, b_0, b_1) = (2, 1, 1, 2)$



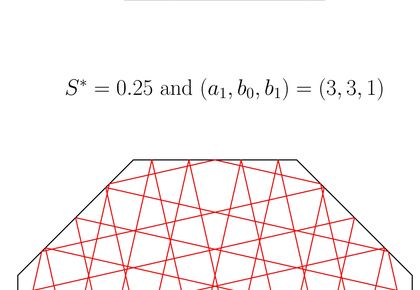
$S^* = 0.5$ and $(a_1, a_2, b_0, b_1) = (4, 3, 2, 2)$



$S^* = 0.25$ and $(a_1, a_2, b_0, b_1) = (3, 2, 1, 1)$



$S^* = 0.1$ and $(a_1, a_2, b_0, b_1) = (11, 7, 1, 1)$



$S^* = 0.5$ and $(a_1, a_2, b_0, b_1) = (3, 2, 9, 13)$