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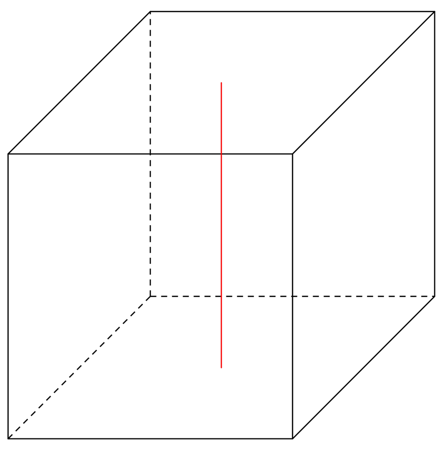
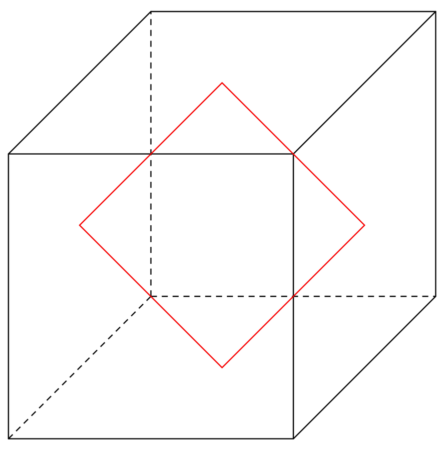
On periodic billiard trajectories in regular polygons and simple closed geodesics on the tetrahedron, cube and octahedron

Periodic billiard trajectories in the cube

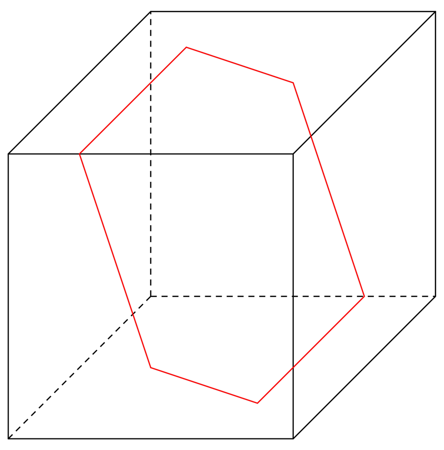
A billiard trajectory in the cube is periodic if and only if for the launching vector with $m, n \in \mathbb{Z}$, $o \in \mathbb{N}_{\neq 0}$ and $u \in \mathbb{R}_{>0}$ the following applies: $\vec{v} = u \cdot \begin{bmatrix} m \\ n \\ o \end{bmatrix}$



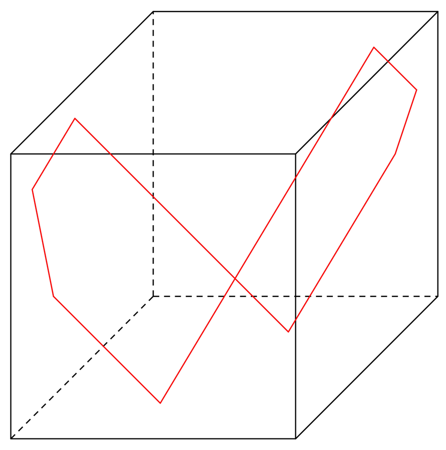
$S = (0.5, 0.5)$ and $\vec{v} = [0; 0; 1]$



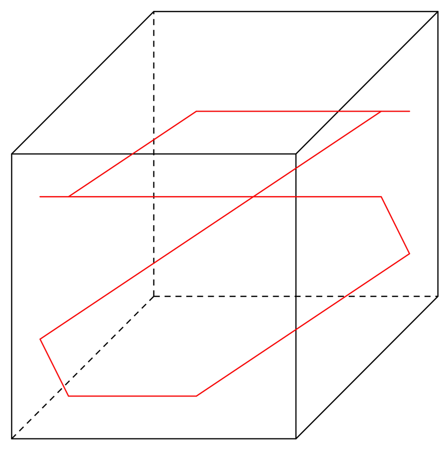
$S = (0.5, 0.5)$ and $\vec{v} = [1; 0; 1]$



$S = (0.75, 0.25)$ and $\vec{v} = [1; 1; 1]$



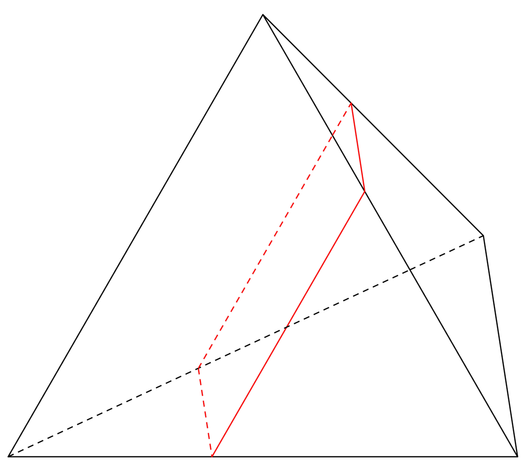
$S = (0.4, 0.25)$ and $\vec{v} = [1; 1; 2]$



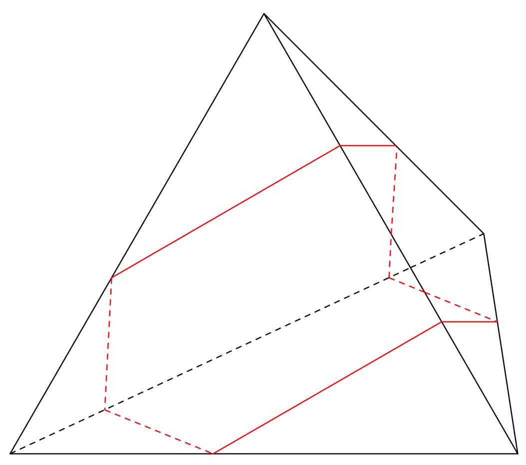
$S = (0.5, 0.3)$ and $\vec{v} = [2; 2; 1]$

Simple closed geodesics on the tetrahedron, cube and octahedron

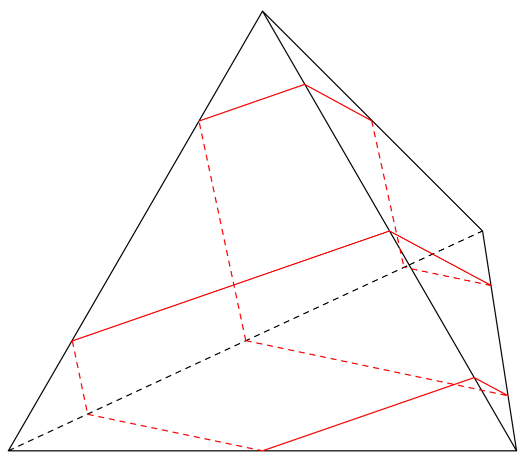
Tetrahedron: A geodesic on the tetrahedron is simple and closed if and only if the launching angle is a product of $\sqrt{3}$ and a rational number.



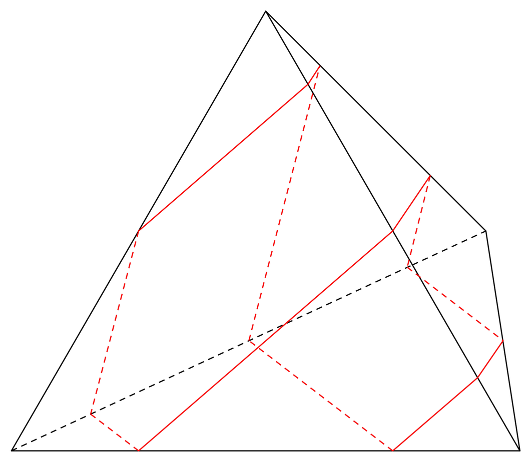
$S = 0.4$ and $\alpha = \frac{\pi}{3}$



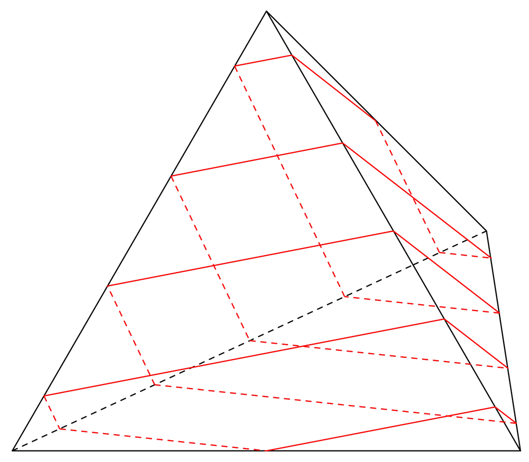
$S = 0.4$ and $\alpha = \frac{\pi}{6}$



$S = 0.5$ and $\alpha = \tan^{-1}(\frac{\sqrt{3}}{5})$



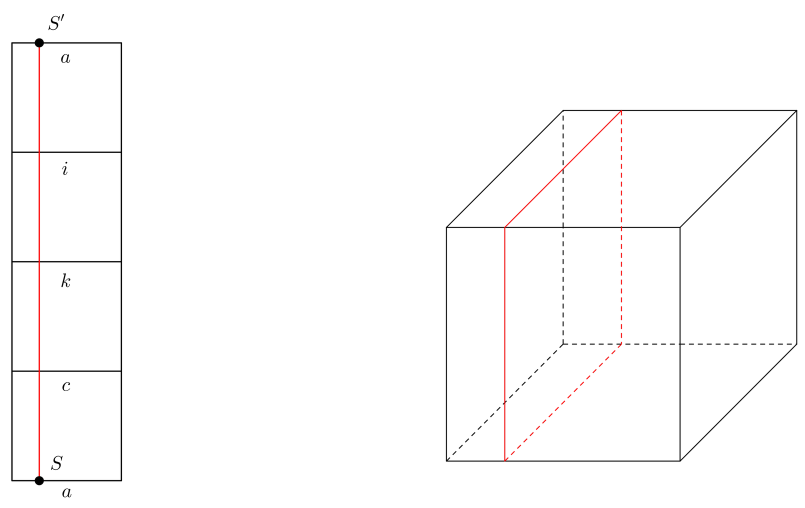
$S = 0.25$ and $\alpha = \tan^{-1}(\frac{\sqrt{3}}{2})$



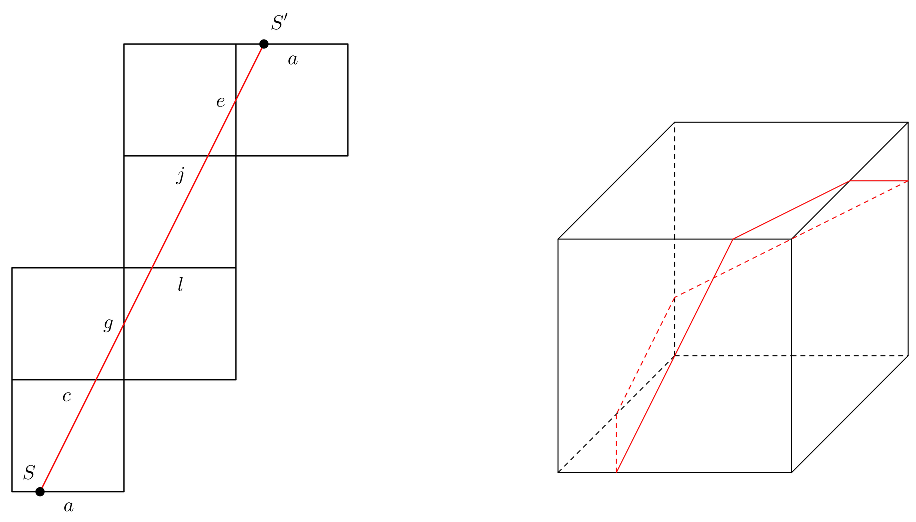
$S = 0.5$ and $\alpha = \tan^{-1}(\frac{\sqrt{3}}{9})$

Cube: A simple closed geodesic on the cube belongs to one of the following classes:

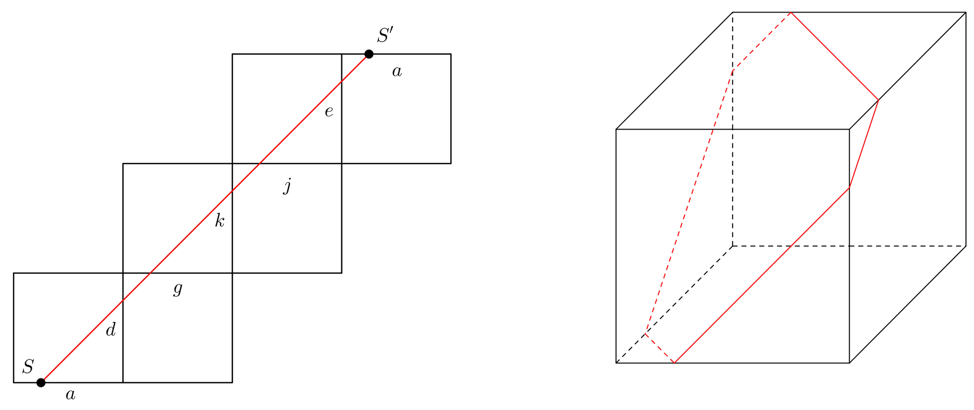
Class	α	Length
(1)	$\frac{\pi}{2}$	4
(2)	$\tan^{-1}(2)$ oder $\tan^{-1}(\frac{1}{2})$	$2\sqrt{5}$
(3)	$\frac{\pi}{4}$	$3\sqrt{2}$



Class (1): $\alpha = \frac{\pi}{2}$ and length of 4



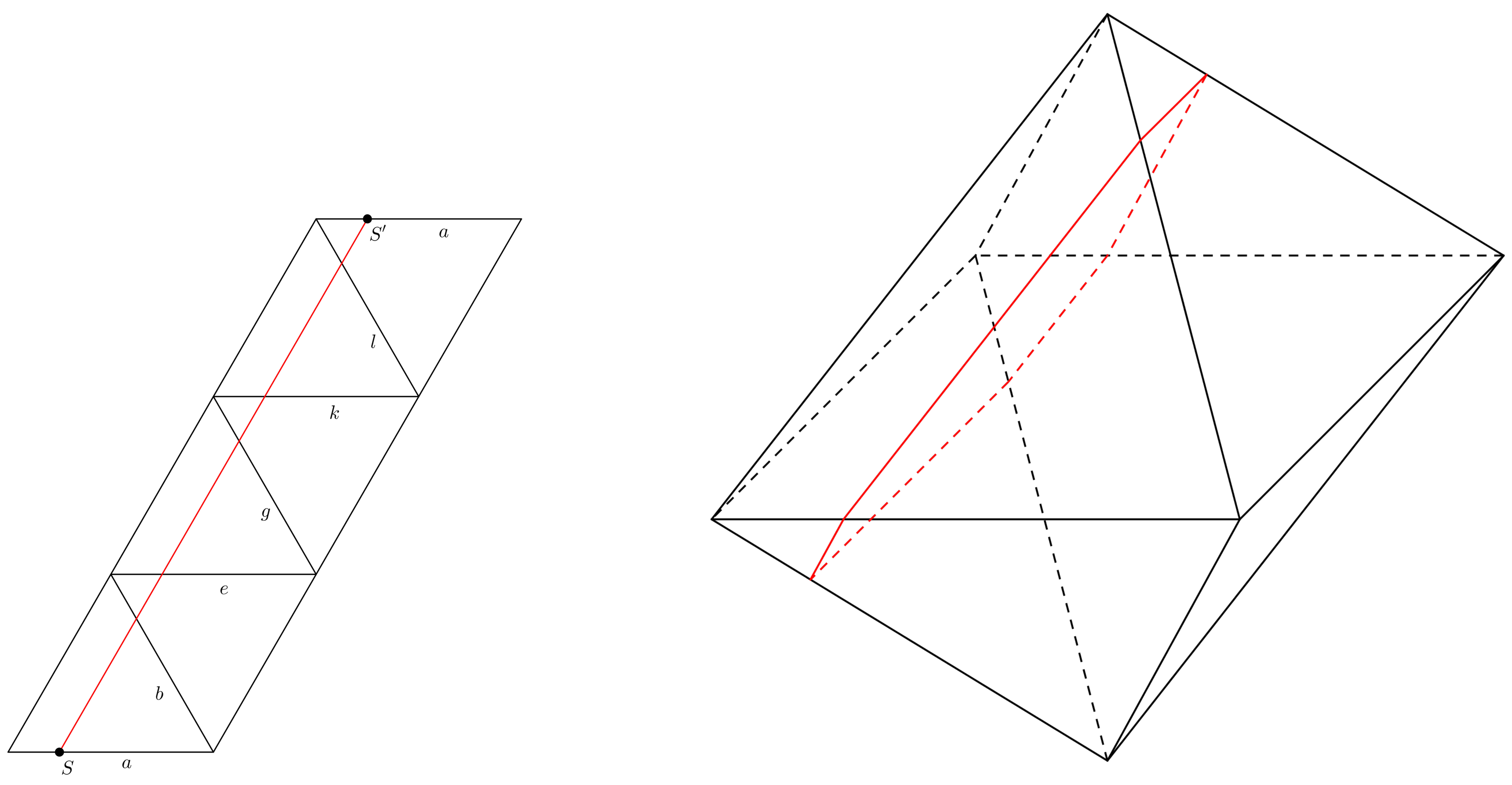
Class (2): $\alpha = \tan^{-1}(2)$ or $\tan^{-1}(\frac{1}{2})$ and length of $2\sqrt{5}$



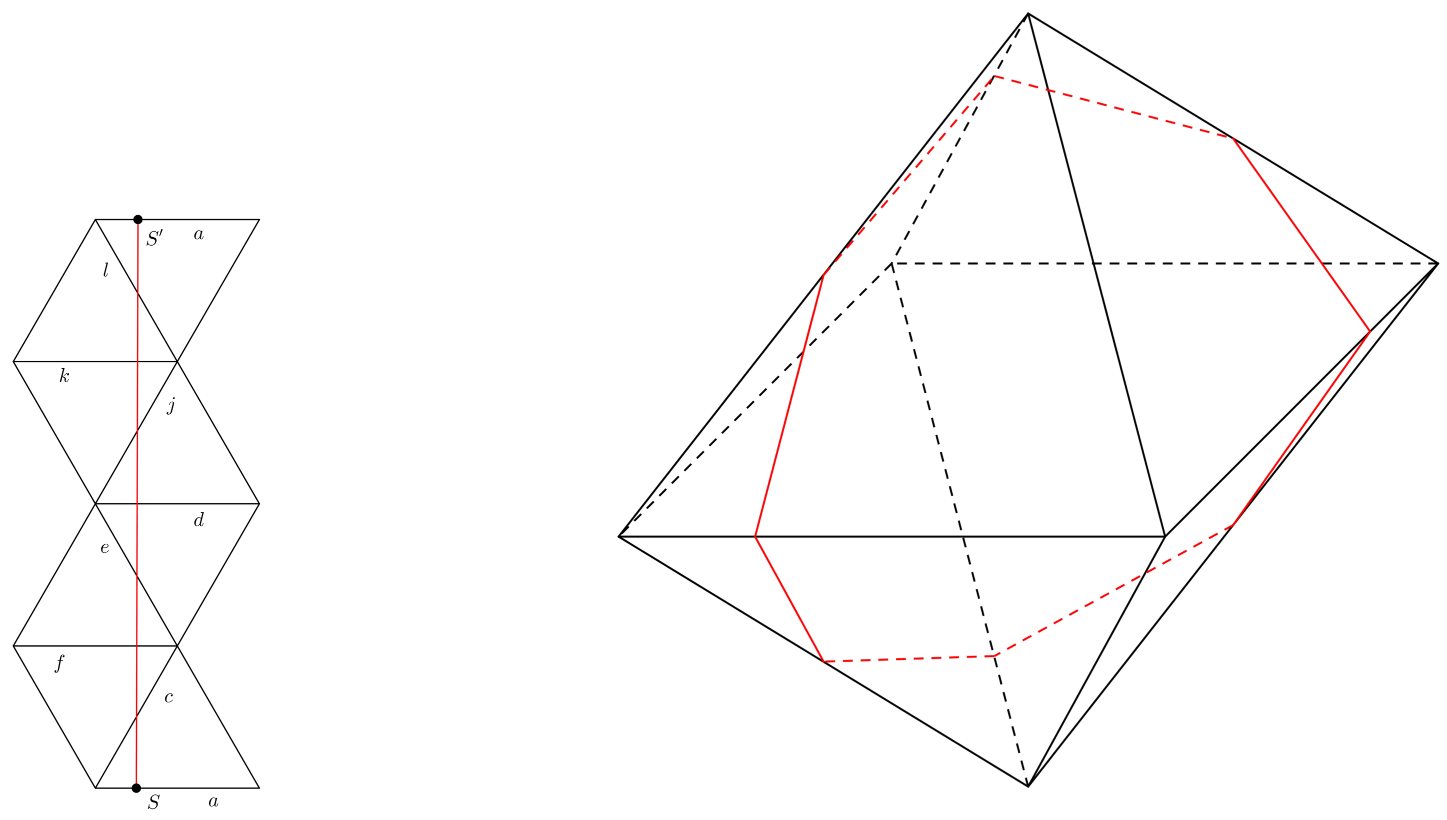
Class (3): $\alpha = \frac{\pi}{4}$ and length of $3\sqrt{2}$

Octahedron: A simple closed geodesic on the octahedron belongs to one of the following classes:

Class	α	Length
(1)	$\frac{\pi}{3}$	3
(2)	$\frac{\pi}{2}$ or $\frac{\pi}{6}$	$2\sqrt{3}$



Class (1): $\alpha = \frac{\pi}{3}$ and length of 3



Class (2): $\frac{\pi}{2}$ or $\frac{\pi}{6}$ and length of $2\sqrt{3}$